Skalenkaskaden in komplexen Systemen

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Multilevel Quadrature in Infinite Dimensions and Applications in Uncertainty Quantification

Abstract:

The coefficients in mathematical models of physical processes are often impossible to determine fully or accurately, and are hence subject to uncertainty. It is of great importance to quantify the uncertainty in the model outputs based on the (uncertain) information that is available on the model inputs. This invariably leads to very high dimensional quadrature problems associated with the computation of statistics of quantities of interest, such as the time it takes a pollutant plume in an uncertain subsurface flow problem to reach the boundary of a safety region or the buckling load of an airplane wing. Higher order methods, such as stochastic Galerkin or polynomial chaos methods, suffer from the curse of dimensionality and when the physical models themselves are complex and computationally costly, they become prohibitively expensive in higher dimensions. Instead, some of the most promising approaches to quantify uncertainties in continuum models are based on Monte Carlo sampling and a model hierarchy. Multilevel Monte Carlo (MLMC) methods have been introduced recently and successfully applied to many model problems, producing significant gains. In this talk I want to recall the classical MLMC method and then show how the gains can be improved further (significantly) by using quasi-Monte Carlo (QMC) sampling rules. More importantly the dimension independence and the improved

gains can be justified rigorously for an important model problem in subsurface flow. To achieve uniform bounds, independent of the dimension, it is necessary to work in infinite dimensions and to study quadrature in sequence spaces. I will present the elements of this new theory for the case of lognormal random coefficients in a diffusion problem and support the theory with numerical experiments.