

## 6 + 1 Compact Course: 26.06. – 10.07.2020

# MATHEMATICAL ANALYSIS OF GEOPHYSICAL MODELS AND DATA ASSIMILATION

### MAIN TOPICS:

- Navier-Stokes and Euler equations
- Global regularity of certain geophysical flows
- Data assimilation algorithm for weather and climate prediction

### OVERVIEW

The basic problem faced in geophysical fluid dynamics is that a mathematical description based only on fundamental physical principles, which are called the "Primitive Equations", is often prohibitively expensive computationally, and hard to study analytically.

In these introductory lectures, aimed towards advanced Master and Graduate Students and PostDocs, Edriss Titi will survey in this intensive mini-course the mathematical theory of the 2D and 3D Navier-Stokes and Euler equations, and stress the main obstacles in proving the global regularity for the 3D case, and the computational challenge in their direct numerical simulations. In addition, the professor of Nonlinear Mathematical Science will emphasize the issues facing the turbulence community in their turbulence closure models.

Taking advantage of certain geophysical balances and situations, such as geostrophic balance and the shallowness of the ocean and atmosphere, he will show how geophysicists derive more simplified models which are easier to study analytically. In particular, E. Titi will prove the global regularity for 3D planetary geophysical models and the Primitive Equations of large scale oceanic and atmospheric dynamics with various kinds of anisotropic viscosity and diffusion.

Moreover, in this course it will also be shown that for a certain class of initial data the solutions of the inviscid 2D and 3D Primitive Equations blowup (develop a singularity) in finite time. Furthermore, the instructor will present some recent results concerning the regularity of certain tropical atmospheric models, and the Primitive equations, coupled to microphysics equations of moisture and clouds formation. Finally, Edriss Titi will also discuss some new algorithms for feedback control and data assimilation for the Navier-Stokes equations and other atmospheric and climate models.



Source: Weizmann Institute of Science

Professor Edriss S. Titi is an internationally recognized expert in applied mathematics, specializing, among other things, in the mathematical theory of fluid dynamics, turbulence and geophysical flows.

Edriss Titi has achieved rigorous mathematical results justifying the equations that govern the planetary scale oceanic and atmospheric dynamics. For a long time, he was able to conclusively clarify open questions of the existence and unambiguousness of solutions to these equations, thus giving the model equations something of a mathematical seal of approval.

### SCHEDULE

Friday	26.06.20, 14:00	Inauguration lecture within MATH+ Friday
Monday	29.06.20, 12:00 – 14:00	Navier-Stokes and Euler equations - a quick survey I
Wednesday	01.07.20, 16:00 – 18:00	Navier-Stokes and Euler equations - a quick survey II
Friday	03.07.20, 14:00 – 16:00	Global regularity of certain geophysical flows I
Monday	06.07.20, 12:00 – 14:00	Global regularity of certain geophysical flows II
Wednesday	08.07.20, 16:00 – 18:00	A data assimilation algorithm for weather and climate prediction I
Friday	10.07.20, 14:00 – 16:00	A data assimilation algorithm for weather and climate prediction II

26.06.2020: Inauguration lecture within MATH+ Friday

## MATHEMATICS OF TURBULENT FLOWS: A MILLION DOLLARS PROBLEM

14:00 – 14:15 Welcome and organizational information  
Prof. Dr.-Ing. Rupert Klein (CRC 1114 Spokesman and member of MATH+ Board)

14:15 – 15:45 Mathematics of Turbulent Flows: A Million Dollars Problem  
Prof. Dr. Edriss Titi

Turbulence is a classical physical phenomenon that has been a great challenge to mathematicians, physicists, engineers and computational scientists. In the end of the last century, chaos theory was developed to explore similar phenomena that occur in a wide range of applied sciences, but the eyes have always been on the big ball – Turbulence. Controlling and identifying the onset of turbulence have a great economic and industrial impact ranging from reducing the drag on cars and commercial airplanes to better design of fuel engines, weather and climate predictions.

It is widely accepted by the scientific community that turbulent flows are governed by the Navier-Stokes equations, for large Reynolds numbers, i.e. when the nonlinear advective effects dominate the linear viscous effects (internal friction within the fluids) in the Navier-Stokes equations. As such, the Navier-Stokes equations form the main building block in any fluid model, in particular in global climate models. Whether the solutions to the three-dimensional Navier-Stokes equations remain smooth, indefinitely in time, is one of the most challenging mathematical problems.

Therefore, by the turn of the millennium, it was identified by the Clay Institute of Mathematics as one of the seven most outstanding Millennium Problems in mathematics, and it has set one million US dollars prize for solving it. Notably, reliable computer simulations of turbulent flows is way out of reach even for the most powerful state-of-the-art supercomputers. In this talk the main challenges that the different scientific communities are facing while attempting to attack this problem are described using layman language. In particular, the mathematical point of view of turbulence will be emphasized.

## REGISTRATION

The course is open to all advanced Master and PhD students as well as PostDocs of the CRC 1114 and BMS. It will be held in English and is free of charge. It is recommended to participate on all days.

To register, please send an email to Kristine Al Zoukra at [gs-sfb1114@lists.fu-berlin.de](mailto:gs-sfb1114@lists.fu-berlin.de) by May 29<sup>th</sup> 2020.

## MODE OF COURSE

Due to the current extraordinary circumstances, the course will held entirely online. Details will be announced in due time.

## FUNDED BY

The logo for the Deutsche Forschungsgemeinschaft (DFG), consisting of the letters 'DFG' in a bold, blue, sans-serif font.The logo for MATH+, featuring the text 'MATH+' in a bold, blue, sans-serif font with a green plus sign, and 'Berlin Mathematics Research Center' in a smaller font above it.

### THE EINSTEIN FOUNDATION

Edriss Titi is an Einstein Fellow in the Collaborative Research Center (CRC 1114) "Scaling Cascades in Complex Systems" at Freie Universität Berlin. His work focuses on the rigorous analysis of manifold scale interactions in the atmosphere and oceans. He also develops novel methods for so-called data assimilation, which can be used to integrate the diversity of currently available observational data on weather and climate events into computer-based calculation models in the best possible way.

The Einstein Foundation Berlin was founded in 2009 by the State of Berlin. The Foundation aims to promote science and research of top international calibre in Berlin and to establish the city as a centre of scientific excellence. In addition to its endowment, the Foundation also receives state funding. An independent scientific commission of the highest standard selects projects for funding.