Skalenkaskaden in komplexen Systemen

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Numerical homogenization and the exponential decay of the fine-scale Green's function

Abstract:

This talk presents a variational approach for the numerical homogenization of elliptic partial differential equations with arbitrary rough diffusion coefficients. The trial and test space in this (Petrov-)Galerkin method are derived from linear finite elements on a coarse mesh of width H by local fine-scale correction. The correction is based on the pre-computation of $O(H^{-d})$ independent cell problems on patches of diameter H log(1/H). The moderate overlap of the patches suffices to prove O(H) convergence of the method without any scale-dependent pre-asymptotic or resonance effects. The key step in the error analysis is the proof of the exponential decay of the so-called fine-scale Green's function, i.e., the impulse response of the variational equation in the absence of coarse-scale finite element functions. Among further applications of the approach are the acceleration of solvers for non-linear eigenvalue problems as well as pollution-free high-frequency scattering.